

On the Infinite in Leibniz's Philosophy

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Abstract

This research project on the place of the infinite in Leibniz's philosophy aims to map out the various fields of knowledge through which Leibniz related to the infinite; to identify the connections between them and to present the function of the infinite within Leibniz's broader philosophical theory. To this end, analysis has been made of Leibniz's mathematical approach towards the infinite, the development of his attitude towards the physical continuum and his treatment of various subjects in which the infinite has an essential role, such as the concept of God, the world's status as a whole and as an aggregate and the Principle of Individuation of substance. The general picture that emerges is one where the concept of the infinite applied by Leibniz has a dual meaning, quantitatively and qualitatively, as has been suggested by Nachtomy (2005). The quantitative aspect of the infinite refers to a multitude which can never be considered as a whole, whereas the qualitative aspect of the infinite indicates an indivisible unity. Leibniz tried to combine these two different aspects of the infinite in his philosophy, but he was only partially successful.

In order to provide the intellectual background for Leibniz's attitude to the concept of the infinite, the first part of the research project surveys the different approaches consolidated by thinkers who preceded Leibniz. Among those who had dealt with the concept of the infinite, the following have been included: Aristotle, Thomas Aquinas, William of Ockham, Rabbi Abraham Cohen Herrera, Galilei Galileo, René Descartes, Pierre Gassendi and Baruch Spinoza. Common to almost all these thinkers is the tension between unity and plurality, which emerges directly from their analysis of the notion of the infinite.

The second part of this research project discusses the mathematical aspect of the infinite in Leibniz and includes detailed reference to his Infinitesimal Calculus, developed during his sojourn in Paris between the years 1672-1676. For the duration of this period, Leibniz ignored the problem of the tension between the infinite parts and the whole continuum and made a fundamental assumption that this tension was subject to resolution. Leibniz focused on exposing the opposing connection between summation and differentiation, a connection that gained exposure through his work on infinite series,

theoretically defined in his geometrical works and clearly expressed in the algebraic symbols that Leibniz formulated. However, during mid 1676, Leibniz reached the conclusion that the differential (from whatever order) can not be an actual magnitude. The differential can not exist as an actual component within the continuum and an infinite quantity of it can never generate a whole. Once Leibniz reached the conclusion that the Infinitesimal was a mathematical fiction, he completed the last step towards an entirely syncategorematic approach towards the infinite. The concluding chapter of this part deals with a conceptual clarification of Leibniz's notion of the quantitative infinite and with its justification in light of later approaches to the subject, such as that of Cantor.

The mathematical aspect of the infinite has a direct repercussion on Leibniz's theory of the continuum and indeed, on his physics. The third part of research assesses five different theories formulated by Leibniz throughout his career, with the aim of solving the Labyrinth of the Continuum. Two of these theories were formulated by Leibniz in the years preceding the Infinitesimal Calculus; the third emerged from the new mathematical discoveries made in Paris. However, once he realized the fictitious nature of the infinitesimal, Leibniz drew the conclusion proposed by a fourth theory, according to which, an actual continuum is not possible at all, since there is a severance between an ideal continuum lacking particles and infinitely real discrete particles. Only the discovery of force in the foundation of matter, made it possible for Leibniz finally to modify his conclusions and give recognition to the existence of an actual continuum. The discovery of force enabled Leibniz to identify the constant essence of matter which prevents it from diminishing into dust within infinite division. Moreover, with force in hand, Leibniz was able to make use of the fictitious infinitesimals, in spite of the gap between the real and the ideal. Now he was able to claim that all changes of matter are continuous and lacking in leaps because they are imaginary, without making matter itself an imaginary phenomena. Leibniz was able to do this by making use of the fictitious infinitesimal, describing the imaginary change of the actual reality and even explaining how it could end – even though it can never end. Therefore, absolute rest, for example, is not possible according to Leibniz's final theory. All this accords with Leibniz's Law of Continuity, which "entails that with respect to continuous things, one can treat an external extremum as if it were internal, so that the last case or instance, even if it is of a nature completely

different, is subsumed under the general law governing the others."¹ The discovery of force conservation enabled Leibniz to formulate his important Law of Continuity and made possible his later famous philosophical development of the Individual Substance.

The last part of research is devoted to an analysis of several of Leibniz's philosophical treatises wherein the infinite plays a central role. This part is divided into three sections - God, Worlds and Individuals. Discussion about the concept of God as the most perfect being reveals that Leibniz's quantitative notion of the infinite is insufficient in that it emphasizes the infinite plurality that is emanated from the first cause and never refers to the absolute and undivided unity that characterizes God. The same conclusion emerges from the discussion of the Principle of Individuation of substances. The mathematical aspect of the infinite appropriately expresses the infinite multitude of predicates and relations that have been included in the complete concept of an individual, but inadequately describes the unity that organizes that individual.

In these kinds of cases, Leibniz needs to use a qualitative, metaphysical conception of the infinite, which does not express quantity in any way. God, for example, is the absolute infinite, completely simple and inaccessible to any investigation. The individual substance, also, is defined first and foremost as an infinite being, though limited in relation to God. Leibniz's usage of the qualitative aspect of the infinite is not limited to metaphysics only. As mentioned before, the solution of the physical Labyrinth of the Continuum is dependant upon the existence of an internal force, which Leibniz calls 'infinite', and which generates all changes in the material body of the corporeal substance.

Along with this, the mathematical aspect of the infinite is still very important since a purely qualitative notion of the infinite is insufficient. In the discussion about the world and its pre-established harmony, it seems that the two aspects of the concept of the infinite, compliment each other. On the one hand, the permanent and absolute immensity of God, expressed in the actual world, is translated into infinite points of perspective that are expressed according to an organized spatial and completely ideal plan. On the other hand, these infinite points of view are the perspectives of the metaphysical monads which, in effect, constitute the world as an actual aggregate. Thus, the study of the

¹ *Acta Erud.* 1713 (Supplementa), *Epistola ad V. Cl. Christianum Wolfium, circa Scientiam Infiniti*; GM V 365, trans. by Grosholz 2007, 205.

infinite teaches that the world according to Leibniz is a complete, ideal program whilst at the same time, being an incomplete actual aggregate. Indeed, the metaphysical Law of Continuity that provided a solution to the Labyrinth of the Continuum, already pointed towards a connection between an infinite multitude of states and the metaphysical unity that generates them. However, this law is only appropriate for an organic unity such as that of corporeal substance, whereas the world according to Leibniz is not an organic unity in which God plays the role of its soul. Therefore, the two conflicting perspectives of the infinite yield a dualistic picture of the world, which cannot be unified to present a coherent description.

Ultimately, the concept of the infinite also comes to the fore in Leibniz's research into the analytic notion of truth and in the safeguarding of his theory from determinism. Leibniz made an ontological and not only an epistemological severance between God's absolute knowledge of the contingent futurities and human freedom, by means of infinite analysis of contingent truths. Understanding the reciprocal correlation that exists between mathematics, logic and metaphysics in Leibniz's philosophical theory, highlights that Leibniz solved (or could solve) the Labyrinth of Freedom in the same manner in which he resolved the Labyrinth of the Continuum. After all, Leibniz explicitly declared that these two fundamental problems should be dealt with by the same powerful means, namely, the infinite.² I maintain that the fictitious status of the infinitesimal has an important role in resolving these two labyrinths.

Research of the infinite in Leibniz's philosophical system does not only reveal many links between the fields of knowledge that Leibniz dealt with but also the difficulties and the problems raised by the concept of infinity. In this manner, research of the infinite in Leibniz's philosophy is similar to that of his own experience in the understanding of the infinite – it never ends and does not offer a comprehensive and final interpretation of his entire philosophy, but it has a significant function in its clarification.

² "For there are two labyrinths of the human mind, one concerning the composition of the continuum, and the other concerning the nature of freedom, and they arise from the same source, infinity." (1689?, *On Freedom*; FC 180, AG 95).